Critical behaviour of a one-dimensional Hubbard model with 1/sinh hopping

1995 J. Phys.: Condens. Matter 7 2285

(http://iopscience.iop.org/0953-8984/7/11/006)

J. Phys.: Condens. Matter 7 (1995) 2285–2291. Printed in the UK

Critical behaviour of a one-dimensional Hubbard model with 1/sinh hopping

P-A Bares† and F Gebhard†‡

† Institut Laue–Langevin, BP 156X, F-38042 Grenoble Cédex, France
‡ Department of Physics and Materials Sciences Centre, Philipps University Marburg, D-35032 Marburg, Germany

Received 2 December 1994

Abstract. Applying the finite-size scaling in conformal field theory, we evaluate the correlation functions and susceptibilities of a 1D Hubbard model with $1/\sinh(\kappa r)$ hopping and repulsive interaction in an external magnetic field. The error introduced in our approximation scheme should be exponentially small in the thermodynamic limit. Incidentally, we provide the generalization of previous work on critical exponents to the case where the right and left Fermi velocities do not coincide.

Very recently [1] we introduced a one-dimensional $1/\sinh(\kappa r)$ Hubbard model that interpolates between the standard [2] and the $1/r$ Hubbard [3] models. Assuming integrability and applying the asymptotic Bethe *ansatz* to this model, we showed [1] that the Bethe *ansatz* equations yield consistent results for the ground state energy and excitation spectrum in all known solvable limits [4, 5]. At half filling, backscattering is relevant for arbitrary positive U and our model exhibits a Mott transition at $U_c(\kappa > 0) = 0^+$. While its $\kappa \to 0$ limit is singular, it maps onto the $1/r$ Hubbard model which is known to have a $U_c(\kappa = 0) = 2\pi$ [3].

In the present work, we evaluate the asymptotic behaviour of correlation functions for the $1/\sinh(\kappa r)$ Hubbard model by the finite-size scaling in conformal field theory [6, 7]. The model we want to understand resides on an infinite chain while the model we really study has a finite length L and periodic boundary conditions. The rigorous justification of our approach is a delicate issue which we shall not discuss here [8]. We believe that the corrections to the long-distance ($|x| \gg \kappa^{-1}$) long-time behaviour of correlation functions resulting from these approximations are exponentially small in the thermodynamic limit.

Simple yet powerful, bosonization techniques have been a very popular approach to correlation functions in the last decade. Nevertheless, there are real limitations to the bosonization approach. For instance, in the presence of an external magnetic field, the low-energy effective theory cannot be described by n independent Hamiltonians (one for each critical degree of freedom). It is known from previous work (see [7, 9]) that in the presence of a magnetic field the spectrum of scaling dimensions of conformal operators in a model with n internal degrees of freedom is not determined by a single renormalized parameter but rather by the so-called $(n \times n)$ 'dressed charge matrix', essentially the Scattering matrix for density fluctuations at the Fermi surface. Below, we report on a generalization of the results of [6, 7, 9] to the case where the Fermi points are asymmetric and the Fermi velocities do not coincide. Many formulas we derive for the $1/\sinh(\kappa r)$ Hubbard model are natural generalizations of those of [7].

0953-8984/95/112285+07$19.50

Consider a Hubbard model for N spin-$\frac{1}{2}$ electrons on a chain of L sites with hopping amplitude $t(l-m)$, on-site repulsion U, chemical potential μ and magnetic field $h = 2\mu_B H$ (μ_B is the Bohr magneton),

$$\hat{H} = \sum_{l\neq m,\sigma} t(l-m)\hat{c}^{+}_{l,\sigma}\hat{c}_{m,\sigma} + U\sum_{l}\hat{n}_{l,\uparrow}\hat{n}_{l,\downarrow} + \mu\sum_{l,\sigma}\hat{n}_{l,\sigma} - \frac{1}{2}h\sum_{l}(\hat{n}_{l,\uparrow}-\hat{n}_{l,\downarrow}) \tag{1}$$

where $t(l-m) = -\mathrm{i}\sinh(\kappa)(-1)^{l-m}/\sinh[\kappa(l-m)]$ and κ^{-1} controls the effective hopping range. The model whose critical behaviour we want to investigate is given by (1) with $L \to \infty$. It has the odd-parity dispersion curve $\epsilon(k) = -\sinh(\kappa)(\partial/\partial z)\log\Theta_3(z, \mathrm{e}^{-\kappa})\big|_{z=k/2}$, where $\Theta_3(z,q)$ denotes a Jacobi theta function [10]. In [1], we have employed the asymptotic Bethe *ansatz* and have proposed to evaluate the spectrum of excitations of the model (1) by solving the Lieb–Wu-like Bethe *ansatz* equations.

For the sake of generality, it is convenient to rewrite the Bethe *ansatz* equations in the compact form [9]

$$p_\alpha(\lambda^\alpha_j) = p^0_\alpha(\lambda^\alpha_j) + \frac{1}{L}\sum_{\beta=\mathrm{c,s}}\sum_{k=1}^{N_\beta}\phi^0_{\alpha\beta}(\lambda^\alpha_j - \lambda^\beta_k) \tag{2}$$

where $p^0_\alpha(\lambda^\alpha_j)$ denotes the bare momentum given by $p^0_\mathrm{c}(v_j) = H^{-1}(v_j)$ for $\alpha = \mathrm{c}$ and $p^0_\mathrm{s}(\Lambda_j) = 0$ for $\alpha = \mathrm{s}$, while $p_\alpha(\lambda^\alpha_j) = 2\pi I^\alpha_j/L$ is the dressed momentum. The quantum numbers I^α_j, $\alpha = \mathrm{c,s}$ ($I^\alpha_- < I^\alpha_j < I^\alpha_+$) are integers or half-odd integers according to: $I^c_j = N_s/2 \bmod 1$ and $I^s_j = (N_c - N_s + 1)/2 \bmod 1$, where N_α denotes the number of pseudoparticles of type $\alpha = \mathrm{c,s}$. $H^{-1}(v)$ is the inverse of $v = H(k)$ with the convention that $p^0_\mathrm{c}(v^-_j) < p^0_\mathrm{c}(v^+_j)$ if $I^{c,-}_j < I^{c,+}_j$. It can be obtained from the two-body scattering problem as

$$H(k) = \left[-\eta(k) + \epsilon(k)[\epsilon(k) - U]/2\sinh(\kappa)\right]/2 \tag{3a}$$

where

$$\eta(k) = \eta(-k) = (1-q^2)\frac{\partial}{\partial q}\log\Theta_3\left(\frac{k}{2\pi}, q\right)\Big|_{q=\mathrm{e}^{-\kappa}}. \tag{3b}$$

$\phi^0_\alpha(\lambda^\alpha_j - \lambda^\beta_k)$ denotes the bare scattering matrix: $\phi^0_{\mathrm{cc}}(x) = 0$, $\phi^0_{\mathrm{cs}}(x) = \phi^0_{\mathrm{sc}}(x) = -\theta(2x)$ and $\phi^0_{\mathrm{ss}}(x) = \theta(x)$, with $\theta(x) = -2\tan^{-1}(2x/U)$. In this notation, the energy and momentum are given, respectively, by

$$E = \sum_{\alpha=\mathrm{c,s}}\sum_{j=1}^{N_\alpha}\epsilon^0_\alpha(\lambda^\alpha_j) \tag{4a}$$

$$P = \sum_{\alpha=\mathrm{c,s}}\sum_{j=1}^{N_\alpha}p_\alpha(\lambda^\alpha_j) \tag{4b}$$

where $\epsilon^0_\mathrm{c}(v) = \mu - h/2 + \epsilon(H^{-1}(v))$ and $\epsilon^0_\mathrm{s}(\Lambda) = h$.

In order to calculate the finite-size corrections to the energy and momentum, it is convenient to introduce the scattering matrix for the density fluctuations

$$\Phi_{\alpha\beta}(\lambda|\mu) = \phi^0_{\alpha\beta}(\lambda-\mu) + \sum_{\delta=\mathrm{c,s}}\int_{q^-_\delta}^{q^+_\delta} K_{\alpha\delta}(\lambda-\lambda')\Phi_{\delta\beta}(\lambda'|\mu) \tag{5}$$

where $K_{\alpha\beta}(x) = (\partial/\partial x)\phi^0_{\alpha\beta}(x)$, and $q^\pm_\alpha$ denote the right (left) Fermi momenta of the α-type pseudoparticle, i.e., $q^-_\alpha < \lambda^\alpha_j < q^+_\alpha$. For a given magnetic field and chemical potential, $q^\pm_\alpha$

are determined from the conditions that $\epsilon_\alpha(q_\alpha^\pm|q^\pm) = 0$, where the dressed energies obey the integral equations (compare with [6, 7, 9])

$$\epsilon_\alpha(\lambda|q^\pm) = \epsilon_\alpha^0(\lambda) + \sum_\beta \int_{q_\beta^-}^{q_\beta^+} \frac{d\lambda'}{2\pi} \epsilon_\beta(\lambda'|q^\pm) K_{\beta\alpha}(\lambda' - \lambda). \tag{6}$$

We further define the 'dressed charge matrices'

$$Z_{N\alpha\beta}^\pm = \delta_{\alpha\beta} \pm \frac{1}{2\pi}\left[\Phi_{\alpha\beta}(q_\alpha^+|q_\beta^\pm) + \Phi_{\alpha\beta}(q_\alpha^-|q_\beta^\pm)\right] \tag{7a}$$

$$Z_{J\alpha\beta}^\pm = \delta_{\alpha\beta} + \frac{1}{2\pi}\left[\Phi_{\alpha\beta}(q_\alpha^+|q_\beta^\pm) - \Phi_{\alpha\beta}(q_\alpha^-|q_\beta^\pm)\right]. \tag{7b}$$

Following [6, 7], we take the large-L limit in the expressions for the energy and momentum while keeping the terms that scale as L^{-1}. Away from half filling, we find for the ground state energy, the excitation spectrum and the momentum, respectively,

$$E_0 - E_0^\infty = -\frac{\pi}{12L}\sum_{\alpha=\mathrm{c,s}}\left[v_\alpha^+ - v_\alpha^-\right] + O(L^{-2}) \tag{8a}$$

$$\begin{aligned} E(\Delta N, \Delta D) - E_0 = \frac{2\pi}{L}\Bigg[&\frac{1}{8}\Delta N^{\mathrm{T}}\left[V_{NN}^+ - V_{NN}^-\right]\Delta N + \frac{1}{2}\Delta D^{\mathrm{T}}\left[V_{JJ}^+ - V_{JJ}^-\right]\Delta D \\ &+ \frac{1}{4}\Delta D^{\mathrm{T}}\left[V_{JN}^+ + V_{NJ}^-\right]\Delta N + \frac{1}{4}\Delta N^{\mathrm{T}}\left[V_{NJ}^+ + V_{JN}^-\right]\Delta D \\ &+ \sum_{\alpha=\mathrm{c,s}}\left[v_\alpha^+ N_\alpha^+ - v_\alpha^- N_\alpha^-\right]\Bigg] + O(L^{-2}) \end{aligned} \tag{8b}$$

$$\begin{aligned} P(\Delta N, \Delta D) - P_0 = \frac{2\pi}{L}&\left[\Delta N^{\mathrm{T}}\Delta D + \sum_{\alpha=\mathrm{c,s}}(N_\alpha^+ - N_\alpha^-)\right] \\ &+ \sum_{\alpha=\mathrm{c,s}}\frac{1}{2}\left[\kappa_{\mathrm{F}\alpha}^+ + \kappa_{\mathrm{F}\alpha}^-\right]\Delta N_\alpha + \sum_{\alpha=\mathrm{c,s}}\left[\kappa_{\mathrm{F}\alpha}^+ - \kappa_{\mathrm{F}\alpha}^-\right]\Delta D_\alpha + O(L^{-2}) \end{aligned} \tag{8c}$$

where $v_\alpha^\pm$ denote the right (left) Fermi velocities at the respective Fermi points and $V^\pm = \mathrm{diag}(v_\mathrm{c}^\pm, v_\mathrm{s}^\pm)$ is the bare velocity matrix. $D_\alpha = \frac{1}{2}[I_+^\alpha + I_-^\alpha]$ counts the number of α pseudoparticles in excess on the right Fermi point with respect to the left Fermi point. In the lowest-energy state $D_\alpha^0 \neq 0$ ($q_\alpha^- \neq q_\alpha^+$) because of the asymmetry of the dispersion curve. The vectors $\Delta N = N - N^0$, $\Delta D = D - D^0$ and $N_\alpha^\pm$ (particle–hole) characterize the excited states. We use the notation $\kappa_{\mathrm{F}\alpha}^\pm = (2\pi/L)I_\pm^{\alpha,0}$ and $2k_{\mathrm{F}\alpha} = \kappa_{\mathrm{F}\alpha}^+ - \kappa_{\mathrm{F}\alpha}^-$ with $k_{\mathrm{F}\alpha} = \pi N_\alpha/2L$, for $\alpha = \mathrm{c, s}$. The spectral velocity matrices $V_{AB}^\pm$ with $A, B = J, N$ generalize Haldane's velocities $v_{N,J}$ [11] to the case with internal degrees of freedom and where the Fermi points are asymmetric: $V_{AB}^\pm = [(Z_A^\pm)^{-1}]^{\mathrm{T}} V^\pm (Z_B^\pm)^{-1}$.

The finite-size corrections to the ground state energy (8a) tell us that the low-energy effective theory consists of the semidirect product of two Virasoro algebras with central charge $c = 1$. This implies that the low-temperature specific heat is linear in T [12], i.e.,

$$C_v = \frac{k_\mathrm{B}^2\pi}{6}\left(\sum_{a=\pm,\alpha=\mathrm{c,s}}\frac{1}{|v_\alpha^a|}\right)T. \tag{9}$$

Alternatively, the free energy can be evaluated following the ideas of Yang and Yang [13] which also leads to equation (9).

Furthermore, we can calculate the spectrum of conformal dimensions by a simple generalization of the results of [9]. In the present model, the correlation functions for the primary fields are given by (in Euclidean space)

$$\langle\phi_{\Delta_\pm}(x,t)\phi_{\Delta_\pm}(0,0)\rangle = \frac{\exp\left(\mathrm{i}[\Delta N_\mathrm{c}\kappa_{\mathrm{Fc}}^{\mathrm{cm}}+\Delta N_\mathrm{s}\kappa_{\mathrm{Fs}}^{\mathrm{cm}}]\right)\exp\left(-2\mathrm{i}[\Delta D_\mathrm{c}k_{\mathrm{F}\uparrow}+(\Delta D_\mathrm{c}+\Delta D_\mathrm{s})k_{\mathrm{F}\downarrow}]\right)}{(x-\mathrm{i}v_\mathrm{s}^+t)^{2\Delta_\mathrm{s}^+}(x+\mathrm{i}v_\mathrm{s}^-t)^{2\Delta_\mathrm{s}^-}(x-\mathrm{i}v_\mathrm{c}^+t)^{2\Delta_\mathrm{c}^+}(x+\mathrm{i}v_\mathrm{c}^-t)^{2\Delta_\mathrm{c}^-}} \tag{10}$$

where $\Delta_{\mathrm{c,s}}^{\pm}$ denote the scaling dimensions of the primary fields, $k_{\mathrm{Fc}} = k_{\mathrm{F}\uparrow} + k_{\mathrm{F}\downarrow}$, $k_{\mathrm{Fs}} = k_{\mathrm{F}\downarrow}$ with $k_{\mathrm{F}\sigma} = (\pi/2)(N_\sigma/L)$, for $\sigma = \uparrow, \downarrow$, and $\kappa_{\mathrm{F}\alpha}^{\mathrm{cm}} = (\kappa_{\mathrm{F}\alpha}^{+} + \kappa_{\mathrm{F}\alpha}^{-})/2$ is the centre of mass momentum of the Fermi sea of α pseudoparticles. The correlation functions for the physical operators are linear combinations of the above expression. As in the symmetric case, the low-temperature exponential decay of correlation functions can be obtained by a conformal mapping of the entire complex plane onto a strip of width $1/T$ [14]. The conformal dimensions and spins are calculated by comparing the expressions for the energy and momentum finite-size corrections that follow from the general principles of conformal field theory and those in (8*b*) and (8*c*). Since the results are similar to those of [7], we shall omit the explicit expressions (see [8]) and discuss for brevity the exponents in zero magnetic field. As in the standard Hubbard model, the scaling spins are independent of the coupling constant for $h = 0$. The single-particle propagator has k_F, $3k_\mathrm{F}$, ... oscillations with respective exponents: $2\Delta_\mathrm{c}^{\pm} = \vartheta^{\pm}/32 + 1/(2\vartheta^{\pm}) \pm \frac{1}{4}$, $2\Delta_\mathrm{s}^{\pm} = \frac{1}{4} \pm \frac{1}{4}$ and $2\Delta_\mathrm{c}^{\pm} = 9/32\vartheta^{\pm} + 1/(2\vartheta^{\pm}) \pm \frac{3}{4}$, $2\Delta_\mathrm{s}^{\pm} = \frac{1}{4} \pm \frac{1}{4}$, where $\vartheta^{\pm} = 2(\xi^{\pm})^2$ and $\xi^{\pm} = \xi(4q_\mathrm{c}^{\pm}/U) = Z_{\mathrm{cc}}^{\pm}$. It can be shown that $\xi(z)$ obeys the integral equation

$$\xi(z) = 1 + \int_{z_0^-}^{z_0^+} \mathrm{d}z'\, \widetilde{K}(z-z')\xi(z') \tag{11}$$

where $z_0^{\pm} = 4q_\mathrm{c}^{\pm}/U$ and the kernel $\widetilde{K}(z)$ is given by

$$\widetilde{K}(z) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \frac{\mathrm{e}^{-|\omega|}}{2\cosh(\omega)} \mathrm{e}^{\mathrm{i}\omega z}. \tag{12}$$

For $|q_\mathrm{c}^{\pm}| \ll U$, a simple calculation leads to $\xi^{\pm} \approx 1 + (2(q_\mathrm{c}^{+} - q_\mathrm{c}^{-}))/(\pi U)$ while for $|q_\mathrm{c}^{\pm}| \gg U$, the Wiener–Hopf method yields $\xi^{\pm} \approx \sqrt{2}\left[1 - U/(8\pi(q_\mathrm{c}^{+} - q_\mathrm{c}^{-}))\right]$. The momentum distribution shows a weak singularity at k_F, i.e., $\langle n_k\rangle \propto \mathrm{constant} - \mathrm{sgn}(k - k_\mathrm{F})|k - k_\mathrm{F}|^{\nu}$ with $\nu = [\vartheta^{+} + \vartheta^{-}]/32 + [1/\vartheta^{+} + 1/\vartheta^{-} 1]/2$. The density–density correlation function shows $2k_\mathrm{F}$, $4k_\mathrm{F}$, ... singularities like $|k - 2k_\mathrm{F}|^{\gamma_1}$, $|k - 4k_\mathrm{F}|^{\gamma_2}$ with $\gamma_1 = \left[\vartheta^{+} + \vartheta^{-}\right]/8$ and $\gamma_2 = [\vartheta^{+} + \vartheta^{-} - 2]/2$. The spin–spin correlation function has a $2k_\mathrm{F}$ singularity with the same exponent as the $2k_\mathrm{F}$ charge–charge correlation function. The singlet and triplet superconducting correlation functions behave at long wavelengths, i.e., $|k| \ll \kappa$, as $|k|^{\beta}$, where $\beta = 2\left[1/\vartheta^{+} + 1/\vartheta^{-}\right]$. The general expressions for the scaling dimensions in the presence of a magnetic field have to be studied numerically.

The macroscopic properties of the system can be expressed in terms of the microscopic parameters of the model, i.e., critical exponents, as was first done in the scalar case by Haldane [11], and later generalized to the multicomponent case by Frahm and Korepin [7]. We define a susceptibility matrix by $\chi_{\alpha\beta}^{-1} = (\partial^2\widetilde{E})/(\partial N_\alpha \partial N_\beta)$ (whose entries are not equivalent but related to the compressibility and magnetic susceptibility), where $\widetilde{E} = E - \mu + LMh$ denotes the internal energy of the system. Setting $\Delta D_\alpha = 0$ for $\alpha = \mathrm{c, s}$ in (8*b*) and comparing with the expansion of the free energy to second order in the change in the number of particles and magnetization, we have $\chi^{-1} = (\pi/4)\left[V_{NN}^{+} - V_{NN}^{-}\right]$. On substitution of V_{NN} and on account of the definition of the compressibility at fixed magnetization and of the magnetic susceptibility at fixed density, we arrive at

$$\kappa^{-1}\Big|_M = \frac{\pi}{2}n^2 \sum_{a=\pm,\alpha=\mathrm{c,s}} |v_\alpha^a| \left(Z_{Jc\alpha}^a + \frac{Z_{Js\alpha}^a}{2} \right)^2 \tag{13a}$$

$$\chi^{-1}\Big|_n = \frac{\pi}{8} \sum_{a=\pm,\alpha=\mathrm{c,s}} |v_\alpha^a| \left(Z_{Js\alpha}^a\right)^2 \tag{13b}$$

where $n = N/L$ denotes the density of particles, M the magnetization and $\mu_{\mathrm{B}} = 1$. Specializing to zero magnetic field, we find

$$\kappa^{-1}(H=0)\Big|_M = \pi n^2 \left[\frac{v_\mathrm{c}^+}{\vartheta^+} - \frac{v_\mathrm{c}^-}{\vartheta^-} \right] \tag{14a}$$

$$\chi^{-1}\Big|_n (H=0) = \pi \left[\frac{v_\mathrm{s}^+}{(\xi_\mathrm{s}^+)^2} - \frac{v_\mathrm{s}^-}{(\xi_\mathrm{s}^-)^2} \right] \tag{14b}$$

where $\xi_\mathrm{s}^\pm = Z_{Jss}^\pm$. From elementary thermodynamics we know that $\kappa^{-1}|_H = \left((\partial M/\partial H)|_n/(\partial M/\partial H)|_\mu\right)\kappa^{-1}|_M$. It is then readily shown that

$$\kappa\Big|_H = \frac{1}{2\pi n^2} \sum_{a=\pm,\alpha=\mathrm{c,s}} \frac{\left(Z_{Nc\alpha}^a\right)^2}{|v_\alpha^a|} \tag{15a}$$

$$\chi\Big|_\mu = \frac{1}{2\pi} \sum_{a=\pm,\alpha=\mathrm{c,s}} \frac{\left(Z_{Nc\alpha}^a - 2Z_{Ns\alpha}^a\right)^2}{|v_\alpha^a|}. \tag{15b}$$

In zero magnetic field we obtain the simple formulae

$$\kappa\Big|_H (H=0) = \frac{1}{4\pi n^2} \left[\frac{\vartheta^+}{v_\mathrm{c}^+} - \frac{\vartheta^-}{v_\mathrm{c}^-} \right] \tag{16a}$$

$$\chi\Big|_\mu (H=0) = \frac{1}{\pi} \left[\frac{1}{v_\mathrm{s}^+} - \frac{1}{v_\mathrm{s}^-} \right] \tag{16b}$$

which have an obvious interpretation in terms of (left–right) average and weighted Fermi surface density of states of the pseudoparticles.

Following Shastry and Sutherland [15], we evaluate the response of the system to twisting the boundary conditions for charge and spin degrees of freedom. The energy increment due to infinitesimally small twisting angles $\phi_{\mathrm{c,s}}$ ($\phi_\mathrm{c} = \phi_\uparrow$ and $\phi_\mathrm{s} = \phi_\downarrow - \phi_\uparrow$) in the α (ground or excited) state is $E^\alpha(\phi_\mathrm{c}, \phi_\mathrm{s}) - E(0,0) = (\pi/L)\Delta\overline{D}_\alpha^{\mathrm{T}} \left[V_J^+ - V_J^-\right] \Delta\overline{D}_\alpha$, where for short we rewrite $V_J^\pm = V_{JJ}^\pm$. For the charge ($\kappa_{\mathrm{Fc}}^+ - \kappa_{\mathrm{Fc}}^-$) process, we have $\Delta\overline{D}_\mathrm{c} = 1 + \phi_\mathrm{c}/(2\pi)$, $\Delta\overline{D}_\mathrm{s} = \phi_\mathrm{s}/(2\pi)$, while for a spin ($\kappa_{\mathrm{Fs}}^+ - \kappa_{\mathrm{Fs}}^-$) process, we need $\Delta\overline{D}_\mathrm{c} = \phi_\mathrm{c}/(2\pi)$, $\Delta\overline{D}_\mathrm{s} = 1 + \phi_\mathrm{s}/(2\pi)$. By definition the charge (spin) $\gamma = \mathrm{c}$ ($\gamma = \mathrm{s}$) current is given by $J_\alpha^\gamma = -\sum_{\sigma=\uparrow,\downarrow} A_\sigma^\gamma (\partial E_\alpha)/(\partial\phi_\sigma)\big|_{\partial\phi_{-\sigma}}$, with $A_{\uparrow,\downarrow}^\mathrm{c} = -|e|$ and $A_\uparrow^\mathrm{s} = -A_\downarrow^\mathrm{s} = \frac{1}{2}$. Using the above results, the charge currents associated with the ($\kappa_{\mathrm{Fc}}^+ - \kappa_{\mathrm{Fc}}^-$) charge (spin) pseudoparticle processes follow as $J_\mathrm{c}^\mathrm{c} = (1/L)\left[V_{Jcc}^+ - V_{Jcc}^-\right]$ ($J_\mathrm{s}^\mathrm{c} = (1/L)\left[V_{Jcs}^+ - V_{Jcs}^-\right]$) while for the spin currents, $J_\mathrm{c}^s = \left[V_{Jcc}^+ - V_{Jcc}^- - 2(V_{Jcs}^+ - V_{Jcs}^-)\right]/(2L)$ ($J_\mathrm{s}^s = \left[V_{Jcs}^+ - V_{Jcs}^- - 2(V_{Jss}^+ - V_{Jss}^-)\right]/(2L)$).

The Drude peak in the charge (spin) DC conductivity depends on the charge (spin) stiffness $\mathrm{Re}\,\sigma_\mathrm{c}(\omega) = 2\pi e^2 D_\mathrm{c}\delta(\omega)$ ($\mathrm{Re}\,\sigma_\mathrm{s}(\omega) = (\pi/2)D_\mathrm{s}\delta(\omega)$). The charge and spin stiffnesses are evaluated from the increment in the ground state energy due to twisting the boundary conditions, $D_\mathrm{c} = \left[V_{Jcc}^+ - V_{Jcc}^-\right]/(4\pi)$, $D_\mathrm{s} = D_\mathrm{c} + \left[V_{Jss}^+ - V_{Jss}^- + V_{Jcs}^+ - V_{Jcs}^-\right]/\pi$

which in zero magnetic field reduce to the simple expressions

$$D_c = \frac{1}{4\pi}\left[2\left(\frac{v_c^+}{\vartheta^+} - \frac{v_c^-}{\vartheta^-}\right) + \frac{1}{2}\left(v_s^+ - v_s^-\right)\right] \tag{17a}$$

$$D_s = D_c + \frac{1}{\pi}\left[v_s^+ - v_s^-\right]. \tag{17b}$$

Near half filling, the charge stiffness is substantially suppressed, i.e., the charge-carrying mass is strongly enhanced, a fact that signals the onset of the insulating state.

As mentioned in the introduction, the half-filled band ($H^{-1}(q_c^\pm) = \pm\pi$) has a gap in the charge spectrum while the spin spectrum is massless, i.e., can be described by Gaussian field theory with central charge $c = 1$. The critical behaviour in the spin sector is controlled by the scalars $Z_{ss}^\pm$. The conformal dimensions are given by $\Delta^\pm = \frac{1}{2}\left[\Delta D_s Z_{ss}^\pm \pm \Delta N_s/2Z_{ss}^\pm\right]^2$. *Due to the* 2π *periodicity of* $H(k)$, *the formulas simplify considerably* [8]. For $h = 0$ ($q_s^\pm = \infty$), closed expressions can be obtained for all physical quantities [1, 8]. At weak magnetic fields, the Wiener Hopf method yields

$$Z_{ss}^\pm(h \to 0) = \frac{1}{\sqrt{2}}\left[1 + \frac{U}{4\pi(q_s^+ - q_s^-)}\right] \tag{18}$$

with $q_s = (U/2\pi)\log(h_0/h)$ and

$$h_0 = 4\left(\frac{2\pi}{e}\right)^{1/2}\int_{-\pi}^{\pi}\frac{dk}{2\pi}H'(k)\epsilon(k)e^{(2\pi/U)H(k)} \tag{19}$$

where $H'(k)$ denotes the first derivative of $H(k)$. The critical field h_c to the ferromagnetic state is ($q_s^\pm = 0$)

$$h_c = -\int_{-\pi}^{\pi}\frac{dk}{2\pi}H'(k)\epsilon(k)\frac{U/2}{(U/4)^2 + H(k)^2} \tag{20}$$

while below h_c, a Taylor expansion leads to

$$Z_{ss}^\pm(h \to h_c^-) \approx 1 - \frac{4\sqrt{h_c - h}}{\pi U f(U;\kappa)} \tag{21}$$

with

$$f(U;\kappa) = -\frac{U}{2}\int_{-\pi}^{\pi}\frac{dk}{2\pi}H'(k)\epsilon(k)\frac{(U/4)^2 - 3H^2(k)}{\left[(U/4)^2 + H^2(k)\right]^3} \tag{22}$$

Note that in both cases and to lowest order in $(q_s^+ - q_s^-)^{-1}$, the integration bounds are symmetric, i.e. $q_s^+ = -q_s^-$.

Many of the results presented in this paper appear to be generalizations of those of [7], yet some are not so obvious. Moreover, the asymmetry of the charge spectrum in the general case makes an analytic approach difficult and substantial numerical work is required. An asymptotic expansion at large κ around $\kappa = \infty$ (Hubbard model) leads to corrections of $\mathcal{O}(e^{-\kappa})$ to the energies. Thus the physical properties of our model map smoothly onto those of the Hubbard model in the $\kappa \to \infty$ limit while the limit $\kappa \to 0$ is very singular. It is remarkable that the application of the asymptotic Bethe *ansatz* [16] reproduces all known results in various limits. Our investigations of the few-body scattering problem indicate that the N-particle wave function of the $1/\sinh(\kappa r)$ Hubbard model has a complicated (compared to Bethe *ansatz*) structure and presumably obeys a recurrence relation that involves the $(N-1)$- and $(N-2)$-body wave functions. This complexity of the eigenfunctions survives in the $\kappa \to 0$ limit where we know [3] that a Mott transition takes place at $U_c > 0$ for a half-filled band.

Acknowledgments

We wish to thank P Nozières for encouragements and discussions. FG thanks P Nozières and his colleagues for the kind hospitality at the ILL during a stay in 1993/94.

References

[1] Bares P-A and Gebhard F 1995 *Europhys. Lett.* at press
[2] Lieb E H and Wu F Y 1968 *Phys. Rev. Lett.* **20** 1445
[3] Gebhard F and Ruckenstein A E 1992 *Phys. Rev. Lett.* **68** 244; 1994 *Phys. Rev.* B **49** 10926
[4] Inozemtsev V I 1990 *Stat. Phys.* **59** 1143; 1992 *Commun. Math. Phys.* **148** 359
[5] Sutherland B, Römer R A and Shastry B S 1994 unpublished
[6] Woynarowich F 1989 *J. Phys. A: Math. Gen.* **22** 4243
[7] Frahm H and Korepin V E 1990 *Phys. Rev.* B **42** 10553; 1991 *Phys. Rev.* B **43** 5653
[8] Bares P-A and Gebhard F unpublished
[9] Izergin A G, Korepin V E and Reshetikhin N Yu 1989 *J. Phys. A: Math. Gen.* **22** 2615
[10] Gradshteyn I S and Ryzhik I M 1980 *Tables of Integrals, Series and Products* (New York: Academic)
[11] Haldane F D M 1981 *J. Phys. C: Solid State Phys.* **14** 2585
[12] Affleck I 1986 *Phys. Rev Lett.* **56** 746
[13] Yang C N and Yang C P 1966 *Phys. Rev.* **150** 321; *Phys. Rev.* **151** 256
[14] Cardy J L 1986 *Nucl. Phys.* B **270** 186
[15] Shastry B S and Sutherland B 1990 *Phys. Rev. Lett.* **65** 243
[16] Sutherland B 1985 *Exactly Solvable Problems in Condensed Matter and Relativistic field Theory (Springer Lecture Notes in Physics 242)* ed B S Shastry, S S Jha and V Singh (New York: Springer); 1971 *J. Math. Phys.* **12** 251